3.2: INVESTIGATIONS OF THE REFLECTION FROM A JUNCTION OF AN IDEAL RECTANGULAR WAVEGUIDE WITH ONE HAVING ROUNDED INSIDE CORNERS

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The magnitude of the reflection coefficient of a simulated joint between a rectangular waveguide having a cross-section with sharp inside corners and one with rounded corners was determined both experimentally and theoretically.

The simulated joint was fabricated in such a manner that all sources of reflection other than the change in the roundness of the inside corners would be negligible. For example, an aluminum mandrel (0.4" x 0.9" x 19") was accurately milled and ground, with each edge rounded to a prescribed radius and was carefully joined to another mandrel identical to the first except for having sharp edges. The assembled mandrel was then electroformed with copper to a substantial thickness. The mandrel was removed chemically, leaving a waveguide shell with inside dimensions duplicating the mandrel. Thus half the length of waveguide had a cross-section with rounded corners and the remaining half had a cross-section with sharp corners. The procedure was followed in fabricating several such simulated joints.

The measurement technique uses reflectometer techniques[1,2] to obtain the magnitude of the scattering coefficient S_{11} of the simulated joint. A directional coupler is oriented to couple to the reflected wave. Appropriately placed tuners are adjusted so that the amplitude $|b_3|$ of the voltage wave from the side arm of the directional coupler is directly proportional to $|\Gamma_L|$ as in Figure 1.

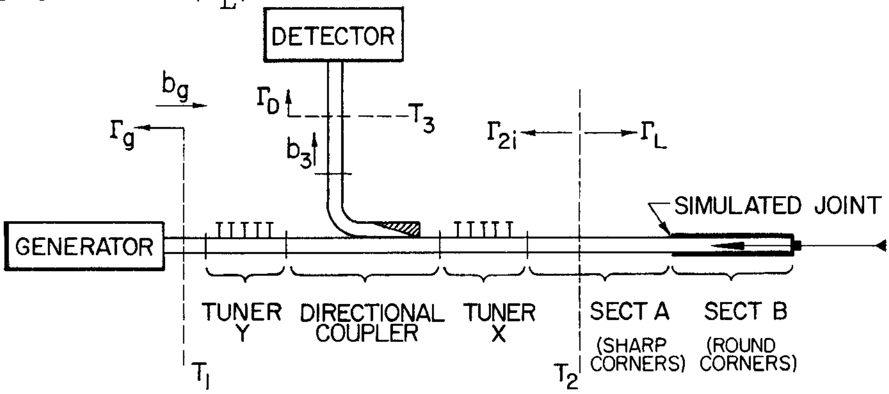

Fig. 1. Diagram of modified reflectometer and simulated joint.

Under these conditions there is no variation in $|b_3|$ as a small reflection load is slid in section A. However when the low reflection load is slid into section B, variations appear and under certain circumstances are superimposed upon a change of level proportional to $|\Gamma_T|$ and $|S_{11}|$. Hence if one knows $|\Gamma_T|$, then $|S_{11}|$ is determinable.

The general expression for the voltage wave from the side arm of a three arm junction is

$$b_3 = C \frac{\frac{1}{K} + \Gamma_L}{1 - \Gamma_{2i} \Gamma_L} .$$

However since $|\Gamma_{2i}|$ and $|\Gamma_L|$ are small and if $|\frac{1}{K}|$ is made small initially, the desired response $|b_3| = |C| \; |\Gamma_L|$ is closely approximated.

The change in level as the load is withdrawn into section B will occur as illustrated in Figure 2 if $|\Gamma_T| > |\frac{1}{K}|$ in section A and $|\Gamma_T| < |\frac{1}{K} + S_{11}|$ in section B. If one were to measure R in db as noted in Figure 2, then

$$R = 20 \log_{10} \frac{|S_{11} + \frac{1}{K}|}{|\Gamma_T|}$$

and the quantity $|S_{11} + \frac{1}{K}|$ is measured in terms of $|\Gamma_T|$.

The results will be similar if the initial adjustments are made in section B rather than in section A. In this case tuner x is adjusted for

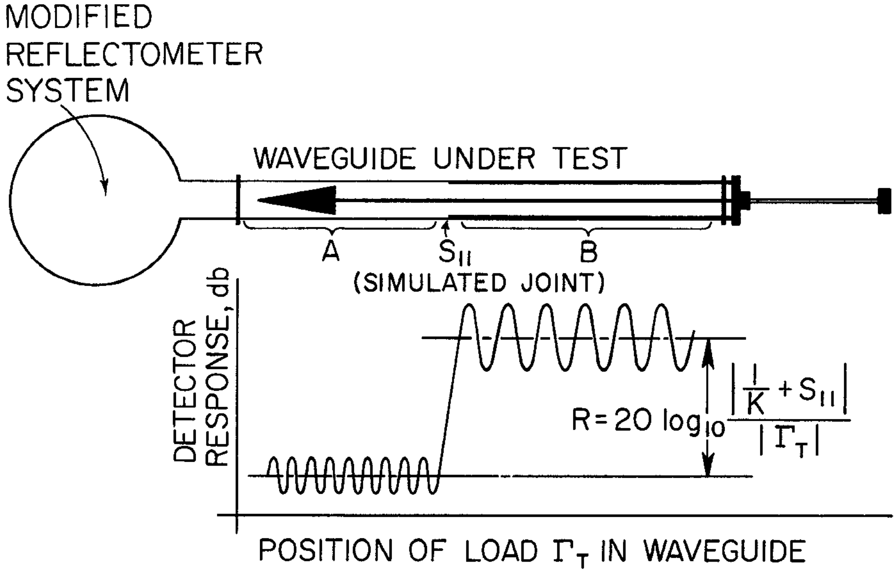

Fig. 2. Detector response when the initial adjustments are made in section A.

no variation in $|b_3|$ as the load is slid in section B. The change in level will then occur when the load is slid into section A. In this case, however, the change will occur if $|\Gamma_T| > |\frac{1}{K} + S_{11}|$ in section B and $|\Gamma_T| < |\frac{1}{K}|$ in section A.

A theoretical formula for the reflection coefficient at the junction of the waveguides was derived by a somewhat indirect method that yields not only the reflection coefficient but also the guide wavelength in the round cornered guide. The compensation theorem, extended to continuous systems by Monteath, is restated for waveguide junctions and approximated for this problem in the form

$$Z' - Z = -1/2 \int_{S'} \mathcal{E}(\underline{h}_a) \times \underline{h}_a \cdot \underline{n}\, dS.$$

Here, Z and Z' are respectively the impedances of the unperturbed and perturbed waveguide junctions, $\underline{h}_a$ is the junction magnetic basis field, $\mathcal{E}(\underline{h}_a)$ is the electric field associated with $\underline{h}_a$ and S' is the complete boundary surface of the perturbation. We then evaluate the integral for the case of an ideal plane termination located in the perturbed guide at an arbitrary distance from the discontinuity, considering the two particular cases corresponding to "open-circuit" and "short-circuit" terminations.

From the above impedances we may obtain the characteristic impedance of the perturbed guide from a well-known theorem of transmission line theory. The reflection coefficient in the rectangular guide at the junction of the two guides is then readily computed and is given by

$$S_{11} = \left(\frac{\lambda_g}{a}\right)^2 \frac{R^2}{ab}\left(\frac{4 - \pi}{8}\right) ,$$

where a and b are the dimensions of the broad and narrow guide walls, λ_g is the (unperturbed) guide wavelength, and R is the effective radius of the fillets, i.e.,

$$R = \sqrt{\frac{R_1^2 + R_2^2 + R_3^2 + R_4^2}{4}} .$$

The error in this expression is believed to be of order R^4. It is interesting to note that a similar result is obtained using the "equal cross-sectional area" concept proposed by S. B. Cohn [3]. A comparison of experimental and theoretical values is shown in Figure 3.

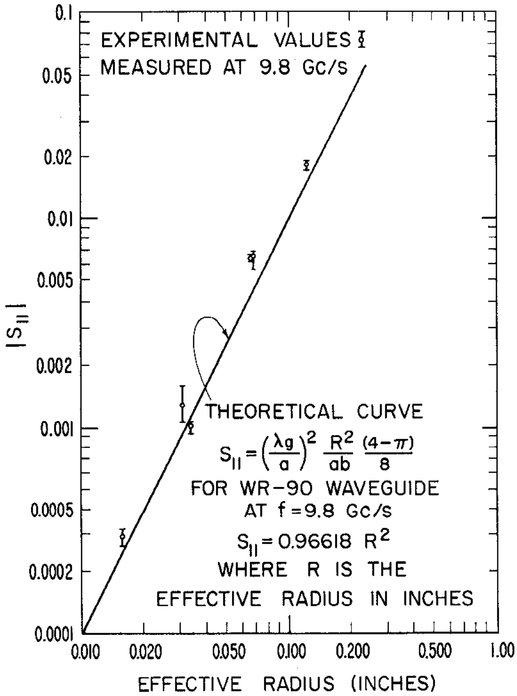

Fig. 3. The reflection from a simulated junction of a rectangular waveguide with one having rounded inside corners.

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1. W. J. Anson, "A Guide to the Use of the Modified Reflectometer Technique of VSWR Measurement," J. Res. NBS 65C, 217-223 (1961).

2. R. W. Beatty, G. F. Engen, and W. J. Anson, "Measurement of Reflections and Losses of Waveguide Joints and Connectors Using Microwave Reflectometer Techniques," Trans. IRE I-9, 219-225 (1960).

3. S. B. Cohn, "Bones from the Technical Graveyard," The Microwave Journal 4, 15-17 (1961).